PIPE DISCHARGE FLOW CALCULATIONS  
(A DIERS Users Group Round-Robin Exercise)

Presented by: 
Joseph C. Leung  
Leung Inc.  
(Consultant to Fauske & Associates, LLC)

Presented at: 
DIERS Users Group Meeting  
Orlando, Florida  
March 23-25, 2009
Review and Update

• Completed nozzle discharge flow calculations for three compositions:
  (I) Cyclohexane (10 bar)
  (II) 20% mole Ethane in Heptane (10 bar)
  (III) 2.5% mole N₂ in Cyclohexane (33 bar)

• Methods used:
  omega method
  PR-EOS flash
  ASPEN Plus & Dynamics
  SIMSCI PRO II
  SuperChem
  VENT (CISP)
**Data Submittal**

Include a summary sheet listing methods.

- Recommend using either $f_{TP} = 0.005$ or Re no. dependent $f_{TP}$.
- Use homogeneous-equilibrium model (HEM).
- Provide, $P$, $T$, $x$ (quality) along the pipe (if available), pipe exit pressure $P_{ex}$, mass flux $G$, and discharge rate $W$ (kg/s) corr. to flow area $A_p$ of 3.355 in$^2$ (2165 mm$^2$).
- E-mail to Joseph Leung (DIERS UG Design/Testing Committee Chair) at leunginc@cox.net or leung@fauske.com.
**Proposed Inlet Conditions**

<table>
<thead>
<tr>
<th>Case</th>
<th>Liquid Composition (mole)</th>
<th>$P_0$ (bar)</th>
<th>$T_0$ (°C)</th>
<th>$x_0$ (vapor mass frac.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ia</td>
<td>100% c-C6</td>
<td>10</td>
<td>182.3</td>
<td>0.0001 (bubble pt)</td>
</tr>
<tr>
<td>Ib</td>
<td>100% c-C6</td>
<td>10</td>
<td>182.3</td>
<td>0.01</td>
</tr>
<tr>
<td>Ic</td>
<td>100% c-C6</td>
<td>10</td>
<td>182.3</td>
<td>0.1</td>
</tr>
<tr>
<td>IIa</td>
<td>20% C2/n-C7</td>
<td>10</td>
<td>51.9</td>
<td>0.0001 (bubble pt)</td>
</tr>
<tr>
<td>IIb</td>
<td>20% C2/n-C7</td>
<td>10</td>
<td>51.9</td>
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<td>10</td>
<td>51.9</td>
<td>0.1</td>
</tr>
<tr>
<td>IIIa</td>
<td>2.5% N₂/c-C6</td>
<td>33</td>
<td>25</td>
<td>0.0001 (bubble pt)</td>
</tr>
<tr>
<td>IIIb</td>
<td>2.5% N₂/c-C6</td>
<td>33</td>
<td>25</td>
<td>0.01</td>
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<td>IIIc</td>
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<td>33</td>
<td>25</td>
<td>0.1</td>
</tr>
</tbody>
</table>
**Horizontal Pipe Discharge Problem**

Same identical inlet (two-phase) conditions as the nozzle case.

- Two different piping (frictional) resistance

<table>
<thead>
<tr>
<th></th>
<th>Pipe I</th>
<th>Pipe II</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.067 in</td>
<td>2.067 in</td>
</tr>
<tr>
<td>L/D</td>
<td>50</td>
<td>225</td>
</tr>
<tr>
<td>L</td>
<td>8.61 ft</td>
<td>38.8 ft</td>
</tr>
<tr>
<td>$K_{en}$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>N</td>
<td>1.5</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Note - $N = K_{en} + 4f_{TP} \frac{L}{D}$, with $f_{TP} = 0.005$, and $K_{en} = 0.5$
Horizontal Pipe Discharge Problem - (Cont'd)

- Alternate use of Reynolds no. dependent $f_{TP}$

$$f_{TP} = \text{function} \left( \frac{GD}{\mu_{TP}} \right)$$

$$N = K_{en} + 4 \bar{f}_{TP} \frac{L}{D}$$

where $\bar{f}_{TP}$ average two – phase friction factor

$$\mu_{TP} = \left[ \frac{x}{\mu_g} + \frac{(1-x)}{\mu_f} \right]^{-1}$$

according to McAdam

$\mu_g, \mu_f =$ vapor and liquid viscosity
Pipe Flow Formulation

- Constant diameter pipe (continuity) –
  \[ G = \rho u = \text{constant} \]

- Energy balance (adiabatic flow) -
  \[ H_1 + \frac{1}{2} G_1^2 v_1 = H_2 + \frac{1}{2} G_2^2 v_2 = \text{constant} \]

- Momentum balance (turbulent flow) -
  \[ v dP + G^2 v dv + \frac{4f}{2D} G^2 v^2 dZ = 0 \]
Expansion Law (Eq. of State)

• Need P-v (pressure - sp. volume) relation.

• Normal practice is to use constant H (enthalpy) flash calculation.

• From adiabatic flow starting from stagnation -

\[ H_o = H + \frac{1}{2} G^2 v \]

a constant H flash assumes K.E. to be small.
Example Illustration

- Vapor cyclohexane discharge through pipe.
- Classical ideal-gas (IG) method:
  - Shapiro text (1953)
  - Bird Stewart Lightfoot text (1960)
  - Levenspiel AIChE J (1977) – Lappel correction
  - Churchill text (1980)
  - Coulson & Richardson text (1996)
- Omega method.
- Constant H analytical integration method.
Classical IG Method

- \( C_p / C_v = k = 1.05 \)
  ideal – gas specific heat values from DIPPR

- \( P – v \) relation (exact)
  \[
  \frac{P}{P_1} = \frac{v_1}{v} \left[ 1 - \left( \frac{k - 1}{2k} \right) \frac{G^2 v_1}{P_1} \left( \left( \frac{v}{v_1} \right)^2 - 1 \right) \right]
  \]

- Momentum equation
  \[
  4f \frac{L}{D} = \frac{k + 1}{k} \ln \left( \frac{v_1}{v_2} \right) + \left[ 1 - \left( \frac{v_1}{v_2} \right)^2 \right] \left( \frac{k - 1}{2K} + \frac{P_1}{G^2 v_1} \right)
  \]
Results from Classical IG Model

IG Density $\rho_{go}^{IG} = 22.3 \text{ kg/m}^3$ (10 bar, 455K)

DIPPR Density $\rho_{go} = 27.6 \text{ kg/m}^3$ ($Z_o = 0.81$)

<table>
<thead>
<tr>
<th></th>
<th>Pipe I</th>
<th>Pipe II</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1.5</td>
<td>5.0</td>
</tr>
<tr>
<td>IG Density</td>
<td>2130 kg/m²s</td>
<td>1560 kg/m²s</td>
</tr>
<tr>
<td>DIPPR Density</td>
<td>2370 kg/m²s</td>
<td>1740 kg/m²s</td>
</tr>
</tbody>
</table>
Omega Method

- $\omega$ parameter at $x_o = 1.0$ is given by

$$\omega = \left(1 - 2 P_o \frac{V_{fgo}}{h_{fgo}}\right) + \rho_{go} C_p T_o P_o \left(\frac{V_{fgo}}{h_{fgo}}\right)^2 = 1.31$$

- Momentum equation ($G^* = G / \sqrt{P_o \rho_o}$)

$$4 f \frac{L}{D} = \frac{2}{G^*^2} \left[\frac{\eta_1 - \eta_2}{1 - \omega} + \frac{\omega}{(1 - \omega)^2} \ln \frac{(1 - \omega)\eta_2 + \omega}{(1 - \omega)\eta_1 + \omega} \right]$$

$$- 2 \ln \left[\frac{(1 - \omega)\eta_2 + \omega}{(1 - \omega)\eta_1 + \omega} \left(\frac{\eta_1}{\eta_2}\right)\right]$$

- Exit choking criterion

$$G_c^* = \frac{\eta_{2c}}{\sqrt{\omega}}$$
**Omega Method**

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<tr>
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</thead>
<tbody>
<tr>
<td>N</td>
<td>1.5</td>
<td>5.0</td>
</tr>
<tr>
<td>G*</td>
<td>0.418</td>
<td>0.311</td>
</tr>
<tr>
<td>G</td>
<td>2190 kg/m²s</td>
<td>1630 kg/m²s</td>
</tr>
<tr>
<td>η₂c</td>
<td>0.478</td>
<td>0.357</td>
</tr>
</tbody>
</table>

Note:  \( P_0 = 10 \) bar, \( \rho_{go} = 27.6 \) kg/m³ (DIPPR)

\( \eta_{2c} \) is exit choking pressure ratio
Constant H Integration Method

- Obtain $P - v$ data from constant H FLASH calculation.

- Fit $P - v$ with best polynomial

$$\frac{v}{v_o} - 1 = a \left( \frac{P_o}{P} - 1 \right) + b \left( \frac{P_o}{P} - 1 \right)^2$$

where $a = 1.38$, $b = 0.012$

- Analytical or numerical integration of differential momentum equation.

$$G^2 = \frac{2 \int_{P_2}^{P_1} \frac{dP}{v}}{4f \frac{L}{D} + 2 \ln \left( \frac{v_2}{v_1} \right)}$$
Cyclo Hexane
10 bara (182.3°C)
vapor inlet Xo=1.0
PR EOS calc

\[ Y = 1.38X + 0.012X^{**2} \]
# Analytical Integration Method

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>1.5</td>
<td>5.0</td>
</tr>
<tr>
<td>$G^*$</td>
<td>0.412</td>
<td>0.307</td>
</tr>
<tr>
<td>$G$</td>
<td>2160 kg/m$^2$s</td>
<td>1610 kg/m$^2$s</td>
</tr>
<tr>
<td>$\eta_{2c}$</td>
<td>0.488</td>
<td>0.366</td>
</tr>
</tbody>
</table>

Note: $P_o = 10$ bar, $\rho_{go} = 27.6$ kg/m$^3$ (DIPPR)
## Summary of c-C6 Vapor Discharge Rate

<table>
<thead>
<tr>
<th></th>
<th>Pipe I (N = 1.5)</th>
<th>Pipe II (N = 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IG (k = 1.05)</td>
<td>4.61 kg/s</td>
<td>3.38 kg/s</td>
</tr>
<tr>
<td>IG w/ real $\rho_{go}$</td>
<td>5.13 kg/s</td>
<td>3.77 kg/s</td>
</tr>
<tr>
<td>$\omega$ method</td>
<td>4.74 kg/s</td>
<td>3.53 kg/s</td>
</tr>
<tr>
<td>Const H analytical</td>
<td>4.68 kg/s</td>
<td>3.48 kg/s</td>
</tr>
<tr>
<td>Average</td>
<td>4.79 kg/s</td>
<td>3.54 kg/s</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.23 kg/s 5%</td>
<td>0.17 kg/s 1.5%</td>
</tr>
</tbody>
</table>
Pipe-Segment Numerical Integration

\[ \Delta L = - \frac{\bar{v} \Delta P + G^2 \bar{v} \Delta v}{2f \frac{G^2}{D} \bar{v}^2} \]

where

- \( \Delta P \) is pressure increment
- \( \Delta v \) is incremental specific volume over \( \Delta P \)
- \( \bar{v} \) is average specific volume in \( \Delta P \)
**Numerical Integration Steps**

1. G is known or guessed.
2. Increments of pressure are taken from the initial to the final pressure.
3. \( \bar{V} \) and \( \Delta v \) are obtained for each increment for a constant-enthalpy process.
4. \( \Delta L \) for each \( \Delta P \) taken is computed from Eq. in previous slide.
5. Total length of pipe \( L \) is \( \sum \Delta L \).
6. If \( \Delta L \) is negative, then \( \Delta P \) is too large.
7. A critical flow condition corresponds to \( \Delta L = 0 \), and the final pressure corresponds to choked pressure.
8. If \( \sum \Delta L > L \), then G was guessed too small and Steps 1-7 are repeated with a larger G. If \( \sum \Delta L < L \), then G was guessed too large; Steps 1-7 are repeated with a smaller G.
9. A converged solution is obtained when \( \sum \Delta L = L \) to within a given tolerance.

*Ref.:* Perry's ChE Hdb, “Fluid Dynamics” section, 7th ed., also Leung, CEP article, 1996.