Risk-Based Approach – Explosions and Structural Response

Introduction to Single Degree of Freedom and Pressure-Impulse Diagrams

An ioMosaic White Paper

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Abstract

This manuscript introduces the Single Degree Of Freedom (SDOF) approach for predicting the response of structures being impacted by an explosion. The concept of pressure-impulse diagrams is introduced and identified as a valuable tool to be used during the analysis of results generated during the development of a risk-based quantitative assessment. The blast loading phenomena, which is critical during the development of the SDOF approach, is fully explained in the following reference:

Dunjó, J; Amorós, M., Prophet, N., Gorski, G., 2016. "Risk-Based Approach – Explosions and Blast Loading. Introduction to Explosions and Structure Blast Loading Phenomena". ioMosaic Corporation

Table of Contents

Appendices

List of Tables

List of Figures

Introduction

The interaction with a structure and a blast wave involves establishing the design of blast loads, setting the structural performance requirements and evaluating the structure to ensure that can withstand these loads within the required performance limits. For practical design purposes and in a risk-based quantitative assessment framework, it is convenient to adopt approximate methods which permit rapid analysis of even complex structures with reasonable accuracy. These methods usually require that both the structure and the loading be idealized in some degree. It is frequently possible to reduce the system to a Single Degree of Freedom (SDOF), a system in which only one type of motion is possible such that the position of the system at any instant can be defined in terms of a single coordinate. Structural elements can be represented by an equivalent SDOF and even though such elements which are parts of a complete structure, it is often permissible to treat them independently **[1]**.

For the idealized system to perform in the same way as an actual structure, it is only necessary to make a proper selection of the system parameters. Transformation factors can be applied to obtain the equivalent mass, spring constant, load and resistance functions. The ideal spring-mass system is selected such that the deflection of the mass is the same at some point of significance on the structure. The key point is that an idealized system which behaves timewise in exactly the same fashion as the actual structure can be constructed and then analyzed with relative ease.

The proposed method should not be regarded as merely a crude approximation. Problems in structural dynamics typically involve significant uncertainties, particularly with regard to loading characteristics. Complex methods of analysis are often not justified and are extremely time-consuming and expensive. It is not practical to employ methods having precision much greater than that of the input of the analysis **[1]**.

This paper proposes a detailed elasto-plastic SDOF numerical dynamic analysis for evaluating each structural element that conforms a blast-loaded closed rectangular building. The proposed methodology maximizes the most reliable criteria by balancing the required level of detail with the inherent uncertainties present in the problem definition. The method of analysis used is optimized because the loading effects of explosions cannot be precisely specified. Based on criteria and assumptions established in referred bibliography below, more detailed efforts would not be justified based on the mentioned inherent uncertainty of the input data analysis.

Single Degree of Freedom (SDOF)

Figure 01 illustrates a SDOF in dynamic equilibrium. By isolating the mass of the SDOF and applying the concept of dynamic equilibrium, the general equation of motion may be written as follows given $\boldsymbol{M}$ as the mass of the system, $\boldsymbol{R}(\boldsymbol{y})$ as the resistance function, $\boldsymbol{c}\dot{\boldsymbol{y}}$ as the damping force, $\boldsymbol{F}(\boldsymbol{t})$ as the load function and $\boldsymbol{y}$, $\dot{\boldsymbol{y}}$ and $\ddot{\boldsymbol{y}}$ as the displacement, velocity and acceleration of the system, respectively.

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{R}(\boldsymbol{y}) + \boldsymbol{c}\dot{\boldsymbol{y}} - \boldsymbol{F}(\boldsymbol{t}) = \boldsymbol{0} \qquad \textbf{E.01}$$

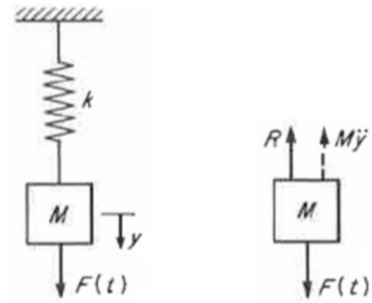

Figure 01: SDOF in Dynamic Equilibrium

Note that the damping force has been considered in Equation **E.01**. While all structural dynamic systems contain damping to some degree, the effect may not be significant if the load duration is short. Several different forms of damping can be considered in structures, but it is generally believed that, for purposes of this analysis, structural damping may be assumed to be viscous. Viscous damping forces are opposite but proportional to the velocity ($\boldsymbol{c}\dot{\boldsymbol{y}}$), where $\boldsymbol{c}$ is a numerical constant which its magnitude is extremely difficult to determine. For this reason, the critical damping concept $\boldsymbol{c_{cr}}$ is introduced, which it is the amount of damping that would eliminate vibration and for a SDOF, it is given by the following equation by defining $\boldsymbol{k}$ as the elastic spring constant of the system (Equation **E.02**):

$$\boldsymbol{c_{cr}} = 2\sqrt{\boldsymbol{kM}} \qquad \textbf{E.02}$$

The concept of critical damping is useful since it is often easier to specify the amount of damping as a certain percentage of critical than it is to arrive at the numerical value for the coefficient $\boldsymbol{c}$. Note that due to the short time in which the structure reaches its maximum response, damping effects have little effect on peak displacements. Taking credit for energy dissipation through viscous damping during the plastic response phase is questionable, which is another reason to ignore damping **[3]**. When considering damping effects it is recommended to not exceed a maximum damping coefficient value greater than 5% of the critical damping coefficient.

Based on the dynamic equilibrium equation of motion of a SDOF (Equation **E.01**), it becomes apparent should focus on defining the following functions:

- The load function loading, $\boldsymbol{F}(\boldsymbol{t})$
- The resistance function, $\boldsymbol{R}(\boldsymbol{y})$

Blast Loading Function

The determination of blast loads on a given structure provides the required information for blast wave characterization. These phenomena are positive and negative peak side-on overpressures and positive and negative phase durations. Once the incident overpressure curve is defined the by a risk-based quantitative assessment, the interaction between the blast wave and the structure should be analyzed. Detailed quantitative criteria for closed rectangular structures loading are strictly based on reference **[4]** and are illustrated in reference **[7].**

The following section of this paper defines an accurate representation of the structure resistance function.

Resistance-Deflection Function

The resistance function for actual structures may have a variety of forms. **Figure 02a [1]** illustrates a typical curve corresponding to a structure of brittle material (Curve A); a typical curve which applies to a structure made of a ductile material with marked yielding such as steel or reinforced concrete (Curve B); and finally represents a situation in which resistance decreases above a certain deflection but before complete failure (Curve C).

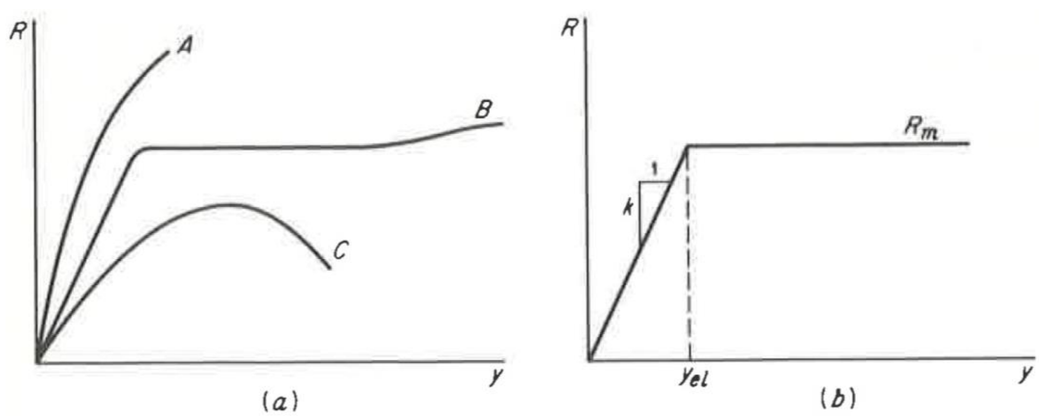

Figure 02: Resistance Functions

While in conventional design, stresses are limited to the elastic range in blast design, yielding is acceptable and is in fact desirable for economic reasons. As the member is stressed in the plastic region, it continues to absorb the blast by balancing the kinetic energy of the explosion against the strain energy of the member. Total strain energy available is a function of dynamic material properties, section properties and the amount of plastic deformation permitted. The total amount of blast energy required to be absorbed is a function of the peak load and duration of the blast wave. Adequacy of a blast loaded member is based on maximum deformation rather than stress level. Elasto-plastic structural behavior is considered for blast loading structural response and the structure resistance-deflection function is a nonlinear function. This elasto-plastic behavior is illustrated and idealized (bilinear function) in **Figure 02b [1]**. The dynamic response of a structure extends through the elastic into the plastic range.

Figure 03b [1] illustrates that as the displacement increases from zero, the resistance increases linearly with a slope of $\boldsymbol{k}$, which is the spring constant. The linearity continues until the elastic limit displacement $\boldsymbol{y_{el}}$ is reached, at which point the maximum spring form $\boldsymbol{R_m}$ has been attained. As the displacement increases further, the resistance is assumed to remain constant at $\boldsymbol{R_m}$. The latter value will be maintained until the ductility limit of the structure $\boldsymbol{\mu}$ is reached, defining the ductility as the ratio of the maximum structure displacement and the yield displacement.

Finally, if the displacement decreases before reaching the ductility limit, the structure is said to "rebound." During the rebound the resistance is assumed to decrease along a line parallel to the initial elastic slope. This decrease will continue with decreasing displacement until a spring force equal to $-\boldsymbol{R_m}$ is attained **[1]**.

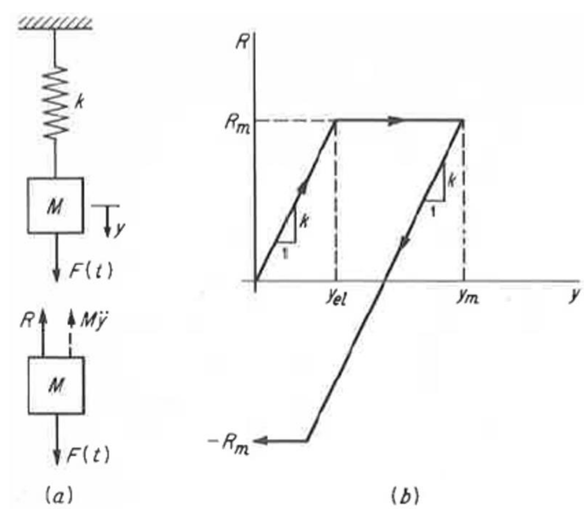

Figure 03: Elasto-Plastic Resistance Function

For an elasto-plastic system, the resistance function is given by Equation **E.03** and equation **E.04.** After the maximum displacement is reached, the dynamic system rebounds and the resistance function is given by Equation **E.05** and Equation **E.06**:

$R = ky$	$y < y_{el}$	**E.03**
$R = R_m$	$y > y_{el}$	**E.04**
$R = k[y_{el} - (y_m - y)]$	$(y_m - y) < 2y_{el}$	**E.05**
$R = -R_m$	$(y_m - y) > 2y_{el}$	**E.06**

Equivalent Elasto-Plastic SDOF

To define an equivalent SDOF, it is necessary to evaluate the parameters of that system, namely: the equivalent mass (M_e), the effective spring constant (k_e) and the equivalent force (F_e). In addition, the load-time function must be established in order to analyze the system. The equivalent system is usually selected so that the deflection of the concentrated mass is the same as that for some significant point on the structure; e.g., the midspan of a beam. It should be noted that stresses and forces in the idealized system are not directly equivalent to the same quantities in the structure. However, knowing the deflection, the stresses in the real structure may be readily computed since the time scale is the same as that of the significant point on the structure.

Transformation Factors

The constants of the equivalent system are evaluated on the basis of an assumed shape of the actual structure. This shape is considered to be the same as the one resulting from the static application of the dynamic loads. It is convenient to introduce certain transformation factors, denoted as K, convert the real system into the equivalent system. When the total load, mass, resistance and stiffness of the real structure are multiplied by the corresponding transformation factors, the parameters for the equivalent SDOF are obtained. The following transformation factors are defined:

- The mass transformation factor, K_M, which is defined as the ratio of equivalent mass to the actual total mass of the structure: $K_M = M_e/M_t$; (Equation **E.07**)
- The load factor, K_L, which is defined as the ratio of equivalent to actual total force: $K_L = F_e/F_t$; (Equation **E.08**)

- The resistance factor, $\boldsymbol{K_R}$, which is defined as the ratio of equivalent to actual maximum resistance, or the ratio of equivalent to actual stiffness; that is: $\boldsymbol{K_R} = \boldsymbol{R_e}/\boldsymbol{R_m}$ and $\boldsymbol{K_R} = \boldsymbol{k_e}/\boldsymbol{k}$; (Equation **E.09**)

- The load-mass factor, $\boldsymbol{K_{LM}}$, which is defined as the ratio of mass and load factors: $\boldsymbol{K_{LM}} = \boldsymbol{K_M}/\boldsymbol{K_L}$; (Equation **E.10**)

Note that the resistance of an element is the internal force tending to restore the element to its unloaded static position. The maximum resistance is the total load having the given distribution which the element could support statically. The stiffness is equal to the total load of the same distribution which would cause a unit deflection at the point where the deflection is equal to that of the equivalent system; i.e., the factor $\boldsymbol{K_R}$, must always equal the factor $\boldsymbol{K_L}$.

While the main intention of the elasto-plastic SDOF is to define an equivalent elasto-plastic resistance function (bilinear), most of the structural elements present three different regions (see **Figure 04 [1]**):

- **Elastic region:** Elastic resistance is the level at which the material reaches yield at the location of maximum moment in the member

- **Elasto-plastic region:** Beyond the point of first yield of a member, plastic regions are formed in the section and an elastic-plastic condition occurs. Internal resistance continues to increase as the stress in other locations of the member rises in response to the applied load, although at a lower slope than the elastic region. During this period, portions of the member are responding plastically while other sections are responding elastically based on cross section and location along the member. As the response continues, other critical sections reach yield and additional plastic hinges are formed. Each yield point changes the slope of the resistance-deflection curve

- **Plastic region:** when the last section yields, no additional resistance is available and the resistance-deflection curve is flat. The area under this curve represents the total strain energy available to resist load at a given deflection

Figure 04 [1] illustrates a typical resistance-deflection curve with elastic, elasto-plastic and plastic regions. This depicts how the function is simplified to be bilinear:

- The effective spring constant $\boldsymbol{k_E}$ is defined so that the areas under the two curves are equal. The energy absorbed will remain constant and there will be little error in the dynamic displacement computed

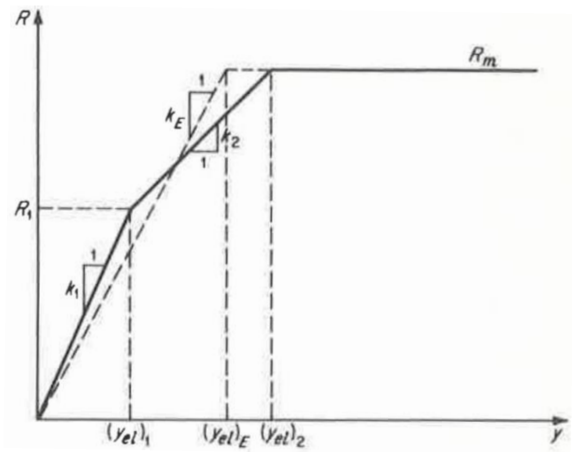

Figure 04: Effective Bilinear Resistance Function

Considering the transformation factors and the definition of the effective bilinear function with rebound, the following four differential equations given below express the equation of motion. The first two equations express the motion before the maximum displacement is reached and the last two equations express the motion when the dynamic system is in elastic rebound:

Applicable equations before reaching $\boldsymbol{y_m}$ (Equation **E.11** and Equation **E.12**):

$$\boldsymbol{K_{LM}M_t\ddot{y} + R(y) + c\dot{y} - F(t) = 0} \qquad \boldsymbol{y < y_{el}} \qquad \textbf{E.11}$$

$$\boldsymbol{K_{LM}M_t\ddot{y} + R_m + c\dot{y} - F(t) = 0} \qquad \boldsymbol{y > y_{el}} \qquad \textbf{E.12}$$

Applicable equations after reaching $\boldsymbol{y_m}$ (Equation **E.13** and Equation **E.14**):

$$\boldsymbol{K_{LM}M_t\ddot{y} + k[y_{el} - (y_m - y)] + c\dot{y} - F(t) = 0} \qquad \boldsymbol{(y_m - y) < 2y_{el}} \qquad \textbf{E.13}$$

$$\boldsymbol{K_{LM}M_t\ddot{y} - R_m + c\dot{y} - F(t) = 0} \qquad \boldsymbol{(y_m - y) > 2y_{el}} \qquad \textbf{E.14}$$

where $\boldsymbol{M_t}$ is the total mass of the beam, slab, or other elements

The natural period and the critical damping coefficient of the system are given by Equation **E.15** and Equation **E.16**, respectively:

$$\boldsymbol{T = 2\pi\sqrt{\frac{K_{LM}M_t}{k}}} \qquad \textbf{E.15}$$

$$\boldsymbol{c_{cr} = 2\sqrt{K_{LM}kM_t}} \qquad \textbf{E.16}$$

Several sources of information provide detailed information on transformation factors to be considered for beams, one-way slabs, two-way, flat slabs and frames. **Appendix I** illustrates transformation factors gathered from references **[1]**, **[2]**, **[3]**, **[4]**, **[5]** and **[6]**.

Dynamic Material Properties and Interaction Design Data

Transformation factors illustrated in **Appendix I** require information of material properties with the aim to estimate ultimate moment capacities, moments of inertia, elasticity and other key parameters that provide information for defining the elasto-plastic resistance function of the structural element. The following contents are intended to introduce the basics of dynamic material properties, associated criteria and key input data correlated to steel properties and concrete.

Further information related to material properties can be found in references **[1]**, **[2]**, **[4]**, **[5]**, **[6]**, **[9]**.

Strength Factors

Strength Increase Factor (SIF)

Static properties are readily available from a variety of sources and are well defined by national codes, standards and organizations. Specifications referenced in the codes define minimum mechanical properties for various grades of material. The average yield strength of steel materials is approximately 25% greater than the specified minimum values. A Strength Increase Factor (SIF) is used to account for this condition and is unrelated to strain rate properties of the material.

Dynamic Increase Factor (DIF)

Construction materials experience an increase in strength under rapidly applied loads. These materials cannot respond at the same rate as the load is applied; the yield strength increases and less plastic deformation will occur. At a fast strain rate, a greater load is required to produce the same deformation than at a lower rate. This increase in the yield stress is quite significant for lower strength materials and decreases as the static yield strength increases. **Figure 05** and reference **[3]** illustrate a typical stress-strain curve describing dynamic and static response of steel (left) and concrete (right).

To incorporate the effect of material strength increase with strain rate, a Dynamic Increase Factor (DIF) is applied to static strength values. DIFs are simply ratios of dynamic material strength to static strength and are a function of material type as well as strain rate. DIFs are also dependent on the type of stress (i.e., flexural, direct shear) because peak values for these stresses occur at different times. Flexural stresses occur very quickly while peak shears may occur relatively late in time resulting in a lower strain rate for shear.

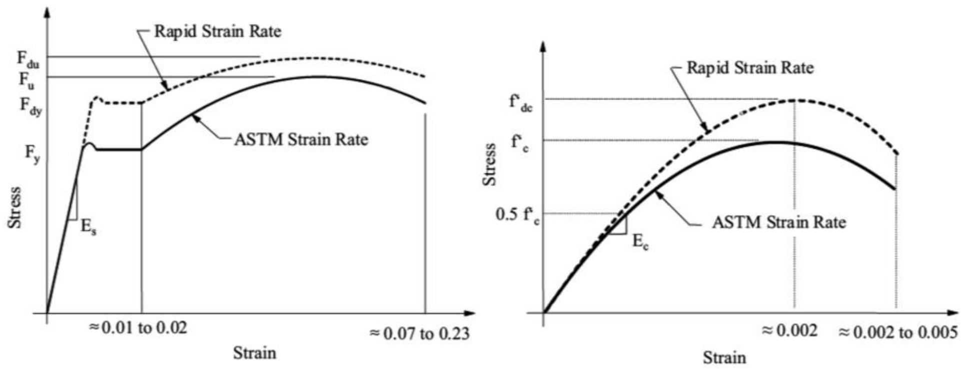

Figure 05: Effect of Strain Rate on Stress-Strain Curve for Steel and Concrete

UFC 3-340-02 **[5]** and other references suggest selecting DIF values based on pressure range or scaled distance to the explosion source. This method group blast loads of less than a few hundred psi (100 psi) into the low pressure category with a single DIF value for each stress type. For petrochemical facilities, the vast majority of structures will fall in this low pressure category.

Stress Factors

Strain hardening effects are modeled in SDOF analysis by using a design stress which is greater than the yield. During dynamic response, the stress level at critical sections in a member varies with strain of the section. In the elastic region, the strain across the section varies with location from the neutral axis of the member. Beyond this region, the member experiences plastic response in which the fiber stress of the entire section exceeds the elastic limit. At this point, the stress is constant over the cross section but is still changing with total member strain.

To predict true dynamic response, it would be necessary to continuously vary the material stress with deformation. This variation is difficult to model using SDOF analysis methods because it requires tracking a complex resistance-deflection curve at each time step. It is easier and desirable to represent the design material stress as a bilinear stress-strain curve in which stress increases linearly with strain to yield and a constant value after yield. This produces a simple, equivalent bilinear resistance-deflection curve which includes strain hardening effects and is relatively easy to incorporate into the SDOF analysis. To achieve this simplification, it is necessary to select a design stress equal to the average stress occurring in the actual response. This can be done by estimating a maximum response range and using recommendations illustrated below. **Table 01** lists key basic nomenclature correlated to dynamic material properties and from **Table 02** to **Table 05** provide specific criteria to be considered.

Table 01: Dynamic Material Properties - Nomenclature

Parameter	Description
$\boldsymbol{SIF}$	Strength Increase Factor
$\boldsymbol{DIF}$	Dynamic Increase Factor
$\boldsymbol{f_y}$	Static yield stress for steel, aluminum or reinforced bars
$\boldsymbol{f_{dy} = f_y \cdot SIF \cdot DIF}$	Dynamic yield stress: steel, aluminum or reinforced bars
$\boldsymbol{f_u}$	Static ultimate stress: steel, aluminum or reinforced bars
$\boldsymbol{f_{du} = f_u \cdot DIF}$	Dynamic ultimate stress: steel, aluminum or reinforced bars
$\boldsymbol{f'_c}$	Concrete static yield stress
$\boldsymbol{f'_{dc} = f'_c \cdot SIF \cdot DIF}$	Concrete dynamic yield stress
$\boldsymbol{f'_m}$	Masonry static yield stress
$\boldsymbol{f'_{dm} = f'_m \cdot SIF \cdot DIF}$	Masonry dynamic yield stress
$\boldsymbol{f_{ds}}$	Dynamic design stress

Table 02: Strength Increase Factors (SIF) for Structural Materials [3]

Structural Material	SIF	
Structural Steel ($\boldsymbol{f_y \leq 50\ ksi;\ f_y \leq 345\ MPa}$)	1.10	
Reinforcing Steel ($\boldsymbol{f_y \leq 60\ ksi;\ f_y \leq 414\ MPa}$)	1.10	
Cold-Formed Steel	1.21	
Concrete, Concrete and Masonry and Other Materials	1.00	

Table 03: DIF for Reinforcing Bars, Concrete and Masonry [3]

Stress Type	DIF			
	Reinforcing Bars		Concrete	Masonry
	(f_{dy}/f_y)	(f_{du}/f_u)	(f'_{dc}/f'_c)	(f'_{dm}/f'_m)
Flexure	1.17	1.05	1.19	1.19
Compression	1.10	1.00	1.12	1.12
Diagonal Tension	1.00	1.00	1.00	1.00
Direct Shear	1.10	1.00	1.10	1.00
Bond	1.17	1.05	1.00	1.00

Table 04: DIF for Structural Steel, Cold-Formed Steel and Aluminum [3]

Structural Material	DIF		
	Yield Stress		Ultimate Stress
	(f_{dy}/f_y)		(f_{du}/f_u)
	Bending / Shear	Tension / Compression	
ASTM A36	1.29	1.19	1.10
ASTM A588	1.19	1.12	1.05
ASTM A514	1.09	1.05	1.00
ASTM A653	1.10	1.10	1.00
ASA AMS5501 (SS)	1.18	1.15	1.00
SAE AMS4113 (Aluminum)	1.02	1.00	1.00

Table 05: Dynamic Design Stress for Structural Steel [3]

Type of Stress	Maximum Ductility	Dynamic Design Stress
All	$\mu \leq 10$	$f_{ds} = f_{dy}$
All	$\mu > 10$	$f_{ds} = f_{dy} + (f_{du} - f_{dy})/4$

Interaction Design Data

Once criteria have been established addressing dynamic material properties, specific information of the material is required for the evaluation of moments of inertia and ultimate moment capacities.

For example, the analysis and design of steel beams under dynamic loading requires the calculation of their resistance in the elastic and plastic ranges of behavior. At points of maximum moment in a beam, yielding due to bending may be assumed to be concentrated and the points will act as plastic hinges. When plastic moments are utilized in resisting blast loads, it is necessary to prevent local and overall buckling of members in order to maintain this plastic bending resistance during distortion. Structural steel details and data are found in the AISC website and AISC Steel Construction Manual **[8]**. For example, second moments of area, elastic section modulus and further calculations can be performed including plastic bending resistance, shear strength and lateral support requirements.

Detailed information on how to evaluate resistance in the elastic and plastic ranges of behavior of structural steel (e.g., beam, columns), reinforced concrete and masonry is out of the scope of this paper and can be bound in references **[1]**, **[4]**, **[5]** and **[6]**.

Once dynamic material properties and interaction design data is fully characterized, the transformation factors illustrated in Appendix I can be evaluated and the resistance-deflection function can be calculated. Therefore, both functions that define the SDOF system are completely characterized. After accounting for the damping function, dynamic calculations can be performed to provide deformation values that can be compared with given and well-established damage criteria.

Structural Response Damage Criteria

Structural Response Limits

Response deformation limits are used to establish that the structure provides adequate protection. These limits are based on the type of structure or component, construction materials, location of the structure and desired protection level.

Blast loaded members reach or exceed yield stresses to achieve an economic design. In general, the more deformation the structure or member is able to undergo without failure, the more blast energy that can be absorbed. As member stresses exceed the yield limit, the stress level is not appropriate for judging member response as is done for static elastic analysis. In dynamic design, the adequacy of the structure is judged on deformation at limit parameters.

Almost all published structural response criteria are presented in terms of parameters which are easily compared with simplified non-linear dynamic response calculations involving one or several degrees of freedom models. These parameters include ductility ratio and hinge rotations, which are based on the peak deflection of the component:

- **Ductility ratio**: Defined as the maximum displacement of the member divided by the displacement at the elastic limit and is commonly designated by the symbol $\boldsymbol{\mu}$. For indeterminate members with multiple plastic hinges, the ductility ratio is typically based on the equivalent yield deflection. The equivalent yield deflection is the ultimate resistance divided by the equivalent elastic stiffness, $\boldsymbol{k_E}$.

- **Hinge rotation**: Defined as a measure of member response which relates maximum deflection of a span and indicates the degree of instability present in critical areas of the member. It is designated by the symbol $\boldsymbol{\theta}$ and is defined in two ways: $\boldsymbol{\theta_1}$, the hinge rotation referring to support rotation and the hinge rotation at center, i.e., $\boldsymbol{\theta_2 = 2\theta_1}$. Note that the response limit values illustrated in Tables below refers to support rotation, $\boldsymbol{\theta_1}$.

Therefore, predicted response can be compared to ductility and support rotation limits.

The values vary with material type, section type and required protection category. For reinforced concrete members, response limits are influenced by the shear reinforcing provided as well as the type of response (i.e., flexure, shear, compression). In general, for elements in which shear or compression are significant, the allowable response is quite low. Where adequate shear capacity is provided, large deflections are permitted.

The deformation limit values listed in **Table 06**, **Table 07** and **Table 08** are based on reference **[3]** and apply for the typical cases where the component is designed by using a procedure that explicitly considers the dynamic component response, such as the SDOF methodology described in this report:

Table 06: Building Damage Levels and Component Response Criteria

Type of Stress	Level	Description
Building	**LOW**	Localized component damage. Building can be used however repairs are required to restore integrity of structural envelope. Total cost of repairs is moderate.
Component		Component has none to slight visible permanent damage.
Building	**MEDIUM**	Widespread component damage. Building should not be occupied until repaired. Total cost of repairs is significant.
Component		Component has some permanent deflection. It is generally repairable, if necessary, although replacement may be more economical and aesthetic.
Building	**HIGH**	Key components may have lost structural integrity and building collapse due to environmental conditions (i.e., wind, snow, rain) may occur. Building should not be occupied. Total cost of repairs approaches replacement cost of building.
Component		Component has not failed, but it has significant permanent deflections causing it to be unrepairable.

Table 07: Response Limits for Steel Components

Component[1]	LOW Response		MEDIUM Response		HIGH Response	
	μ	θ	μ	θ	μ	θ
Hot Rolled Steel Compact Secondary Members (Beams, Girts, Purlins)[2]	3	2	10	6	20	12
Steel Primary Frame Members (with significant compression)[2,3,4]	1.5	1	2	1.5	3	2
Steel Primary Frame Members (without significant compression)[2,3,4]	1.5	1	3	2	6	4
Steel Plates[7]	5	3	10	6	20	12
Open-Web Steel Joists	1	1	2	3	4	6
Cold-Formed Light Gage Steel Panels (with secured ends)[5,8]	1.75	1.25	3	2	6	4
Cold-Formed Light Gage Steel Panels (with unsecured ends)[6,8]	1.0	-	1.8	1.3	3	2
Cold-Formed Light Gage Steel Beams, Girts, Purlins and Non-Compact Secondary Hot RolledMembers[8]	2	1.5	3	3	12	10

Note 1: Response limits are for components responding primarily in flexure unless otherwise noted. Flexure controls when shear resistance is at least 120% of flexural capacity.
Note 2: Primary members are components whose loss would affect several other components supported by that member and whose loss could potentially affect the overall structural stability of the building in the area of loss. Secondary members are those supported by primary framing components.
Note 3: Significant compression is when the axial compressive load is more than 20% of the dynamic axial capacity of the member. Axial compression should be based on the ultimate resistance of the supported members exposed to the blast pressure.
Note 4: Sideway limits for moment-resisting structural steel frames (**H:** Height):

$$\boldsymbol{Low} = \boldsymbol{H}/\mathbf{50},\ \boldsymbol{Medium} = \boldsymbol{H}/\mathbf{30},\ \boldsymbol{High} = \boldsymbol{H}/\mathbf{25}.$$

Note 5: Panels must be attached on both ends with screws or spot welds.
Note 6: Panels are not attached on both ends (for example standing seam roof panels).
Note 7: Steel plate criteria can also be applied to corrugated (crimped) plates if local buckling and other response modes are accounted for in the analysis.
Note 8: Light gage refers to material which is less than 0.125 inches (3 mm) thick.

Table 08: Limits for Reinforced Concrete (RC) and Reinforced Masonry (RM)

Component[1]	LOW Response		MEDIUM Response		HIGH Response	
	μ	θ	μ	θ	μ	θ
RC Beams, Slabs and Wall Panels (no shear reinforcement)	-	1	-	2	-	5
RC Beams, Slabs and Wall Panels (compression face steel reinforcement and shear reinforcement in maximum moment areas)	-	2	-	4	-	6
Reinforced Masonry	-	1	-	2	-	5
RC Walls, Slabs and Columns (in flexure and axial compression load)[2]	-	1	-	2[4]	-	2[4]
RC and RM Shear Walls and Diaphragms	3	-	3	-	3	-
RC and RM Components (shear control, without shear)	1.3	-	1.3	-	1.3	-
RC and RM Components (shear control, with shear)	1.6	-	1.6	-	1.6	-
Pre-stressed Concrete; $(\boldsymbol{w_p \le 0.15})$[3]	1	-	-	1	-	2
Pre-stressed Concrete; $(\boldsymbol{0.15 < w_p < 0.3})$[3]	1	-	$0.25/\boldsymbol{w_p}$	1	$0.29/\boldsymbol{w_p}$	1.5

Note 1: Response limits are for components responding primarily in flexure unless otherwise noted.
Note 2: Applicable when the axial compressive load is more than 20% of the dynamic axial capacity of the member. Axial compression should be based on the ultimate resistance of the supported members exposed to the blast pressure.
Note 3: The reinforcement index, $\boldsymbol{w_p} = (\boldsymbol{A_{ps}/b \cdot d_p})(\boldsymbol{f_{ps}/f'_c})$; where: $\boldsymbol{A_{ps}}$ is the area of pre-stressed reinforcement in tension zone, $\boldsymbol{b}$ is the member width; $\boldsymbol{d_p}$ is the depth to center of pre-stressing steel; $\boldsymbol{f_{ps}}$ is the calculated stress in pre-stressing steel at design load; and $\boldsymbol{f'_c}$ is the concrete compressive strength.
Note 4: A support rotation of 4 degrees is allowed for RC components that have compression face steel reinforcement and shear reinforcement in maximum moment areas.

An example on how to use the structural response damage criteria is illustrated in **Figure 06**. The resistance-deflection profile was generated using the blast loading function. (results illustrated in the left-hand side of **Figure 06**). The elastic SDOF was simulated to estimate the displacement, velocity and acceleration history profiles (results illustrated in the right-hand side in **Figure 06**). The maximum displacement of the structure was estimated to be 0.726 inches, while the displacement at the elastic limit was 0.213 inches resulting in a ductility ratio of 3.41. The associated hinge rotation was 2.31. These key results can be compared to applicable damage criteria to evaluate the structural damage potential.

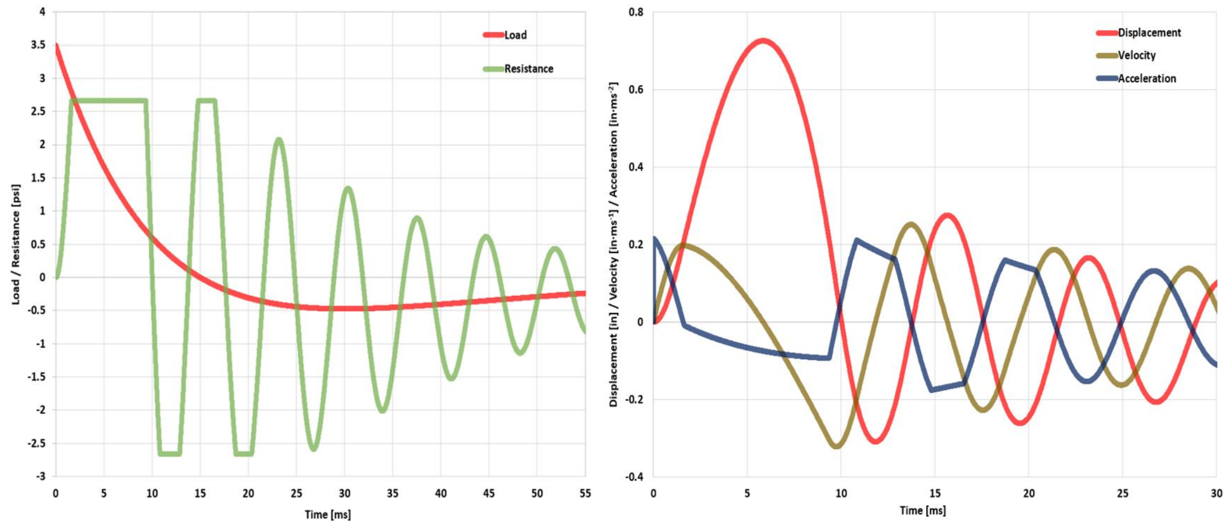

Figure 06: Elasto-Plastic SDOF Example of Dynamic Results

Pressure-Impulse Diagrams

Damage criteria based on ductility ratio can provide further information when an isodamage curve is represented in the space of pressure and impulse of the blast loading **[10]**. The isodamage curve is the well-known Pressure-Impulse (P-I) diagram, which is able to distinguish the damage and undamaged ranges. The P-I diagram was introduced from the analysis of an elastic SDOF model in **[11]**, **[12]**. The P-I diagram has been used widely in damage assessments when structures are subjected to blast loads. For example, the P-I curves with different damage levels have been derived from a study of houses damaged by bombs dropped on UK in the Second World War **[11]**, **[13]**.

These isodamage P-I diagrams have been applied for predicting structural damage and for predicting blast-induced human injuries. Recently, the Department of Defense (DOD) developed criteria for structural response based on P-I diagrams and associated human vulnerability for sixteen (16) different classes of buildings **[14]** using prepopulated P-I diagrams.

The development of P-I diagrams is very useful when analyzing all identified explosions that can impact a given structure. The outputs from a risk-based quantitative analysis may include all blast loads at each side of the impacted structure and the associated frequency of occurrence. Therefore, structural response decision-making can be based on risk values. Structural response and human vulnerability analyses founded on a risk-based quantitative assessment are illustrated in references **[15]**, **[16]**, papers which address buildings and process equipment structural response, respectively.

A P-I diagram with different levels of damage (e.g., different levels of ductility) provides the required information for damage analysis. Computational capabilities allow running multiple iterations for finding a plethora of blast loading functions that satisfy the damage level of interest and for several levels of interest. There are various blast loads (i.e., combinations of pressures and impulses) that produce the same damage level to a given structure. Therefore, given the structure and the damage level of interest, the application of the **E.11** to **E.16** system of equations can provide as many combinations of pressures and impulses as necessary that satisfy the damage level and ensure a complete isodamage curve, for example, at a ductility of 3.

Figure 07 illustrates the P-I diagram that was constructed from the same system as illustrated in **Figure 06**. Several damage levels based on ductility ratio were calculated and the peak overpressure and the associated impulse characterizing the blast load were overlaid. Note that results illustrated in **Figure 06** and **Figure 07** are equivalent and it can be confirmed that the ductility ratio is between 3.0 and 4.0.

Based on **Figure 07**, each isodamage curve is hyperbola-shaped; i.e., contains horizontal and vertical asymptotes. This characteristic confirms that an isodamage P-I diagram is defined by considering the following three regimes:

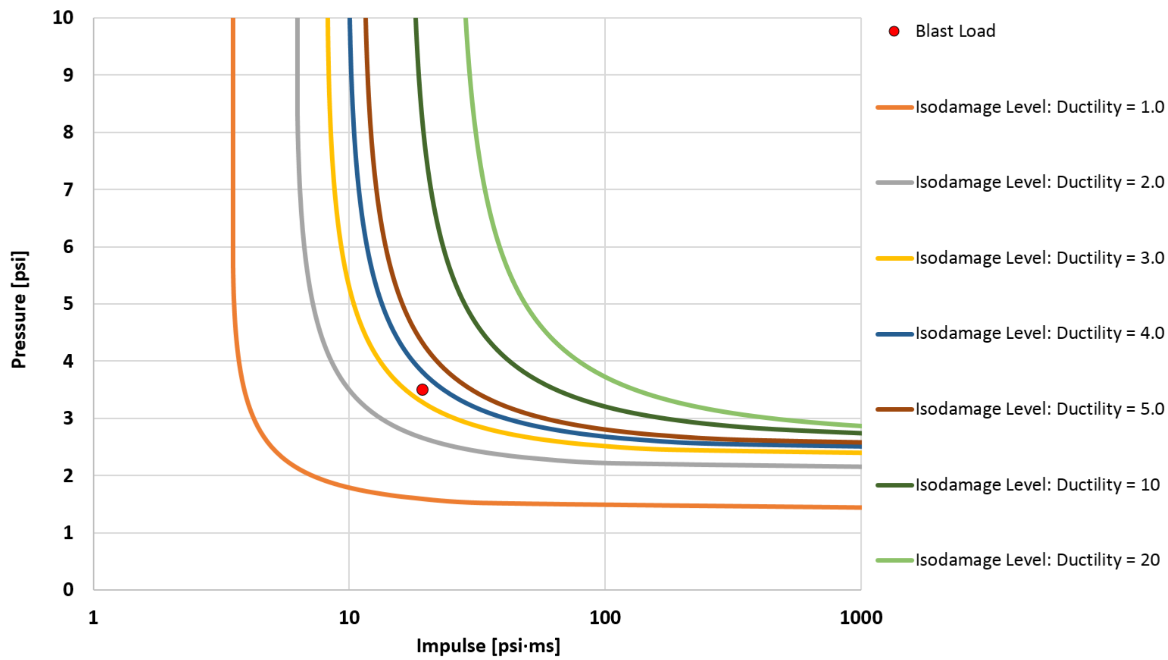

Figure 07: Example of P-I Diagram with Several Damage Levels

- Regime I: Impulse regime; which is controlled by the blast loading impulse only (i.e., vertical asymptote). See **Figure 08** (left-hand side case) **[17]**

- Regime II: Dynamic regime, which is controlled by the combination of pressure and impulse of the blast loading (i.e., intermediate rounded area between asymptotes). See **Figure 08** (center case) **[17]**

- Regime III: Quasi-static regime, which is controlled by the blast loading pressure only (i.e., horizontal asymptote). See **Figure 08** (right-hand side case) **[17]**

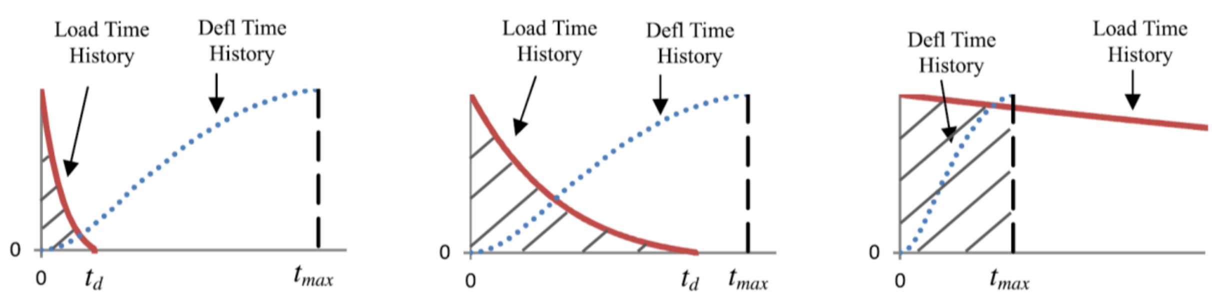

Figure 08: Load-Response Relationships

Conclusions

ioMosaic proposes a detailed elasto-plastic SDOF numerical dynamic analysis for evaluating structures being impacted for explosions. The development of pressure-impulse diagrams has been identified as a valuable tool for evaluating the impact of all explosions identified and characterized during the development of a risk-based quantitative assessment **[18]**.

The proposed methodology maximizes the most reliable criteria as explained in several references above by balancing the required evaluation level of detail with the inherent uncertainties present in the problem definition. This he method is optimized with regard that the loading effects of explosions cannot be precisely specified.

As a summary, it can be concluded that the proposed approach is a non-expensive tool for evaluating explosions affecting target structures and defined during a complete risk-based quantitative assessment.

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[15] Dunjó, J., Amorós, M., Prophet, N., Gorski, G., 2016. “Risk-Based Approach – Facility Siting Addressing Explosions. Impacting Process Plant Permanent and Portable Buildings. Combining Exceedance Curves, Structural Response and Human Vulnerability Criteria”. An ioMosaic White Paper, ioMosaic Corporation.

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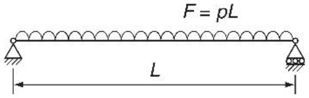
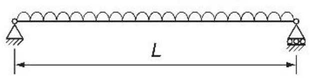
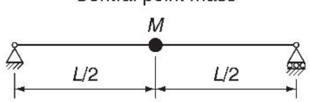
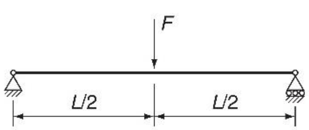
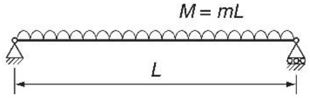
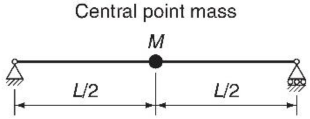
Appendix I: Transformation Factors

Nomenclature and Criteria

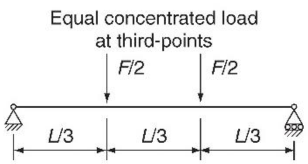
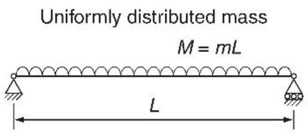
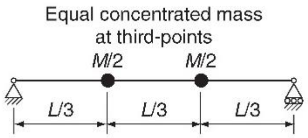
The following contents illustrated in **Appendix I** have been extracted from reference **[6]**, which collected all information from references **[1]**, **[4]**, **[5]** and expanded their contents to cover several additional load cases. Furthermore, nomenclature used in the manuscript is described below:

- d_e: distance from support for calculation of ultimate shear stress
- E: modulus of elasticity
- F: load
- H: Story height
- I: second moment area
- i: specific impulse
- K_L: load transformation factor
- K_M: mass transformation factor
- K_{LM}: load-mass transformation factor
- k_e: equivalent elastic stiffness
- L: span length
- M: mass
- M_p: bending moment resistance
- m: mass per unit length
- p: load per unit length
- R: resistance
- R_m: ultimate resistance
- V: dynamic reaction
- V_S: support shear
- v: ultimate shear stress at distance d_e from support

Equivalent SDOF Factors for Simply Supported Beams

Loading diagram	Mass diagram	Strain range	Load factor, K_L	Mass factor, K_M	Load–mass factor, K_{LM}	Maximum resistance, R_m	Stiffness, k	Support shear, V_s	Ultimate shear stress, v	Dynamic reaction, V
Uniformly distributed load $F = pL$ 	Uniformly distributed mass $M = mL$ 	Elastic	0.64	0.50	0.78	$\frac{8M_p}{L}$	$\frac{384EI}{5L^3}$	$\frac{R_m}{2}$	$R_m\left(\frac{1}{2d_e} - \frac{1}{L}\right)$	$0.39R + 0.11F$
		Plastic	0.50	0.33	0.66	$\frac{8M_p}{L}$	0			$0.38R_m + 0.12F$
	Central point mass M 	Elastic	0.64	1.0	1.56	$\frac{8M_p}{L}$	$\frac{384EI}{5L^3}$	$\frac{R_m}{2}$	$R_m\left(\frac{1}{2d_e} - \frac{1}{L}\right)$	$0.50R$
		Plastic	0.50	1.0	2.0	$\frac{8M_p}{L}$	0			$0.50R_m$
Central point load F 	Uniformly distributed mass $M = mL$ 	Elastic	1.0	0.49	0.49	$\frac{4M_p}{L}$	$\frac{48EI}{L^3}$	$\frac{R_m}{2}$	$\frac{R_m}{2d_e}$	$0.78R - 0.28F$
		Plastic	1.0	0.33	0.33	$\frac{4M_p}{L}$	0			$0.75R_m - 0.25F$
	Central point mass M 	Elastic	1.0	1.0	1.0	$\frac{4M_p}{L}$	$\frac{48EI}{L^3}$	$\frac{R_m}{2}$	$\frac{R_m}{2d_e}$	$0.50R$
		Plastic	1.0	1.0	1.0	$\frac{4M_p}{L}$	0			$0.50R_m$

Equivalent SDOF Factors for Simply Supported Beams (continued)

Loading diagram	Mass diagram	Strain range	Load factor, K_L	Mass factor, K_M	Load–mass factor, K_{LM}	Maximum resistance, R_m	Stiffness, k	Support shear, V_s	Ultimate shear stress, v	Dynamic reaction, V
		Elastic	0.87	0.50[a]	0.58[b]	$\frac{6M_p}{L}$	$\frac{56.4EI}{L^3}$	$\frac{R_m}{2}$	$\frac{R_m}{2d_c}$	$0.525R - 0.025F$
		Plastic	1.0	0.56	0.56	$\frac{6M_p}{L}$	0			$0.52R_m - 0.02F$
		Elastic	0.87	0.76	0.87	$\frac{6M_p}{L}$	$\frac{56.4EI}{L^3}$	$\frac{R_m}{2}$	$\frac{R_m}{2d_c}$	$0.50R$
		Plastic	1.0	1.0	1.0	$\frac{6M_p}{L}$	0			$0.50R_m$

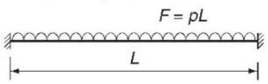
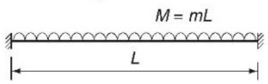
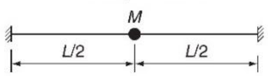
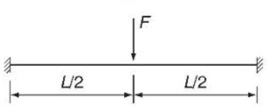
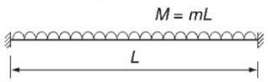
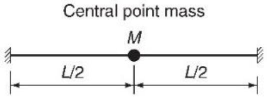
Notation

M_p: Ultimate moment capacity within span

[a] Incorrectly given in reference [1] as 0.52

[b] Incorrectly given in reference [1] as 0.60

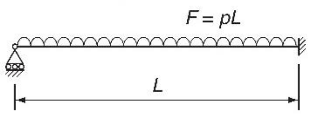
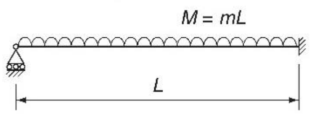
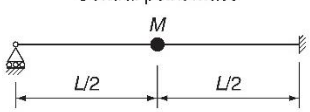
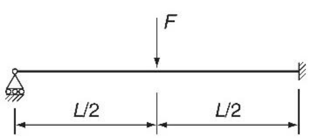
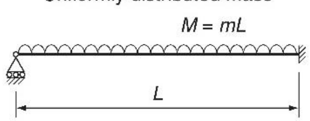
Equivalent SDOF Factors for Fixed Beams

Loading diagram	Mass diagram	Strain range	Load factor, K_L	Mass factor, K_M	Load–mass factor, K_{LM}	Maximum resistance, R_m	Stiffness, k	Equivalent stiffness, k_e[c]	Support shear, V_s	Ultimate shear stress, v	Dynamic reaction, V
Uniformly distributed load 	Uniformly distributed mass 	Elastic	0.53	0.41	0.77	$\frac{12M_{Ps}}{L}$	$\frac{384EI}{L^3}$				$0.36R + 0.14F$
		Elasto-plastic	0.64	0.50	0.78	$\frac{8(M_{ps}+M_{pm})}{L}$	$\frac{384EI}{5L^3}$	$\frac{307EI}{L^3}$	$\frac{R_m}{2}$	$R_m\left(\frac{1}{2d_e}-\frac{1}{L}\right)$	$0.39R + 0.11F$
		Plastic	0.50	0.33	0.66	$\frac{8(M_{ps}+M_{pm})}{L}$	0				$0.38R_m + 0.12F$
	Central point mass 	Elastic	0.53	1.0	1.88	$\frac{12M_{Ps}}{L}$	$\frac{384EI}{L^3}$				$0.50R$
		Elasto-plastic	0.64	1.0	1.56	$\frac{8(M_{ps}+M_{pm})}{L}$	$\frac{384EI}{5L^3}$	$\frac{307EI}{L^3}$	$\frac{R_m}{2}$	$R_m\left(\frac{1}{2d_e}-\frac{1}{L}\right)$	$0.50R$
		Plastic	0.50	1.0	2.0	$\frac{8(M_{ps}+M_{pm})}{L}$	0				$0.50R_m$
Central point load 	Uniformly distributed mass 	Elastic	1.0	0.37	0.37	$\frac{4(M_{ps}+M_{pm})}{L}$	$\frac{192EI}{L^3}$		$\frac{R_m}{2}$	$\frac{R_m}{2d_e}$	$0.71R - 0.21F$
		Plastic	1.0	0.33	0.33	$\frac{4(M_{ps}+M_{pm})}{L}$	0				$0.75R_m - 0.25F$
	Central point mass 	Elastic	1.0	1.0	1.0	$\frac{4(M_{ps}+M_{pm})}{L}$	$\frac{192EI}{L^3}$		$\frac{R_m}{2}$	$\frac{R_m}{2d_e}$	$0.50R$
		Plastic	1.0	1.0	1.0	$\frac{4(M_{ps}+M_{pm})}{L}$	0				$0.50R_m$

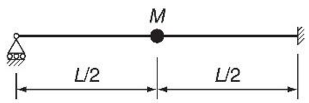
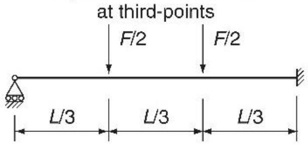
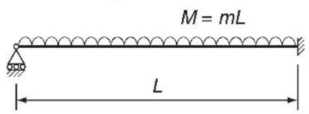
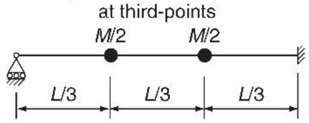
Notation

M_{ps}: Ultimate hogging moment capacity at support
M_{pm}: Ultimate sagging moment capacity within span
Expressions denoted † valid only if $M_{ps} = M_{pm}$
[c] Valid only if $M_{ps} = M_{pm}$

Equivalent SDOF Factors for Propped Cantilevers

Loading diagram	Mass diagram	Strain range	Load factor, K_L	Mass factor, K_M	Load–mass factor, K_{LM}	Maximum resistance, R_m	Stiffness, k	Equivalent stiffness, k_e[c]	Support shear, V_s	Ultimate shear stress, v	Dynamic reaction, V
Uniformly distributed load 	Uniformly distributed mass 	Elastic	0.58	0.45	0.78	$\frac{8M_{ps}}{L}$	$\frac{185EI}{L^3}$	$\frac{160EI}{L^3}$	$V_{s1} = \frac{3R_m}{8}$	$v_1 = R_m\left(\frac{3}{8d_e} - \frac{1}{L}\right)$	$V_1 = 0.26R + 0.12F$ $V_2 = 0.43R + 0.19F$
		Elasto-plastic	0.64	0.50	0.78	$\frac{4(M_{ps} + 2M_{pm})}{L}$	$\frac{384EI}{5L^3}$		$V_{s2} = \frac{5R_m}{8}$	$v_2 = R_m\left(\frac{5}{8d_e} - \frac{1}{L}\right)$	$V = 0.39R + 0.11F \pm M_{Ps}/L$
		Plastic	0.50	0.33	0.66	$\frac{4(M_{ps} + 2M_{pm})}{L}$	0				$V = 0.38R_m + 0.12F \pm M_{Ps}/L$
	Central point mass 	Elastic	0.58	1.0	1.73	$\frac{8M_{Ps}}{L}$	$\frac{185EI}{L^3}$	$\frac{160EI}{L^3}$	$V_{s1} = \frac{3R_m}{8}$	$v_1 = R_m\left(\frac{3}{8d_e} - \frac{1}{L}\right)$	$V_1 = 0.375R$ $V_2 = 0.625R$
		Elasto-plastic	0.64	1.0	1.56	$\frac{4(M_{ps} + 2M_{pm})}{L}$	$\frac{384EI}{5L^3}$		$V_{s2} = \frac{5R_m}{8}$	$v_2 = R_m\left(\frac{5}{8d_e} - \frac{1}{L}\right)$	$V = 0.50R \pm M_{ps}/L$
		Plastic	0.50	1.0	2.0	$\frac{4(M_{ps} + 2M_{pm})}{L}$	0				$V = 0.50R_m \pm M_{ps}/L$
Central point load 	Uniformly distributed mass 	Elastic	1.0	0.43	0.43	$\frac{16M_{ps}}{3L}$	$\frac{107EI}{L^3}$	$\frac{160EI}{L^3}$	$V_{s1} = \frac{5R_m}{16}$	$v_1 = \frac{5R_m}{16d_e}$	$V_1 = 0.25R + 0.07F$ $V_2 = 0.54R + 0.14F$
		Elasto-plastic	1.0	0.49	0.49	$\frac{2(M_{ps} + 2M_{pm})}{L}$	$\frac{48EI}{L^3}$		$V_{s2} = \frac{11R_m}{16}$	$v_2 = \frac{11R_m}{16d_e}$	$V = 0.78R - 0.28F \pm M_{ps}/L$
		Plastic	1.0	0.33	0.33	$\frac{2(M_{ps} + 2M_{pm})}{L}$	0				$V = 0.75R_m - 0.25F \pm M_{ps}/L$

Equivalent SDOF Factors for Propped Cantilevers (continued)

	Central point mass 	Elastic	1.0	1.0	1.0	$\frac{16M_{ps}}{3L}$	$\frac{107EI}{L^3}$	$\frac{160EI}{L^3}$	$V_{s1}=\frac{5R_m}{16}$ $v_1=\frac{5R_m}{16d_e}$	$V_1=0.375R$ $V_2=0.625R$
		Elasto-plastic	1.0	1.0	1.0	$\frac{2(M_{ps}+2M_{pm})}{L}$	$\frac{48EI}{L^3}$		$V_{s2}=\frac{11R_m}{16}$ $v_2=\frac{11R_m}{16d_e}$	$V=0.50R\pm M_{ps}/L$
		Plastic	1.0	1.0	1.0	$\frac{2(M_{ps}+2M_{pm})}{L}$	0			$V=0.50R_m\pm M_{ps}/L$
Equal concentrated load at third-points 	Uniformly distributed mass 	Elastic	0.81	0.45	0.55	$\frac{6M_{ps}}{L}$	$\frac{132EI}{L^3}$	$\frac{122EI}{L^3}$	$V_{s1}=\frac{R_m}{3}$ $v_1=\frac{R_m}{3d_e}$	$V_1=0.17R+0.17F$ $V_2=0.33R+0.33F$
		Elasto-plastic	0.87	0.52	0.60	$\frac{2(M_{ps}+3M_{pm})}{L}$	$\frac{56EI}{L^3}$		$V_{s2}=\frac{2R_m}{3}$ $v_2=\frac{2R_m}{3d_e}$	$V=0.525R-0.025F$ $\pm M_{ps}/L$
		Plastic	1.0	0.56	0.56	$\frac{2(M_{ps}+3M_{pm})}{L}$	0			$V=0.52R_m-0.02F$ $\pm M_{ps}/L$
	Equal concentrated mass at third-points 	Elastic	0.81	0.67	0.83	$\frac{6M_{ps}}{L}$	$\frac{132EI}{L^3}$	$\frac{122EI}{L^3}$	$V_{s1}=\frac{R_m}{3}$ $v_1=\frac{R_m}{3d_e}$	$V_1=0.375R$ $V_2=0.625R$
		Elasto-plastic	0.87	0.76	0.87	$\frac{2(M_{ps}+3M_{pm})}{L}$	$\frac{56EI}{L^3}$		$V_{s2}=\frac{2R_m}{3}$ $v_2=\frac{2R_m}{3d_e}$	$V=0.50R\pm M_{ps}/L$
		Plastic	1.0	1.0	1.0	$\frac{2(M_{ps}+3M_{pm})}{L}$	0			$V=0.50R_m\pm M_{ps}/L$

Notation

M_{ps}: Ultimate hogging moment capacity at support

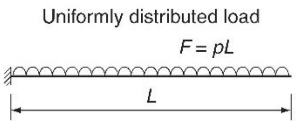
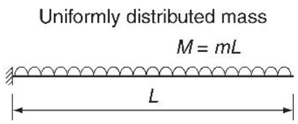
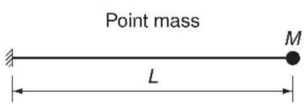
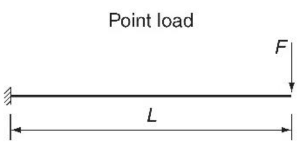
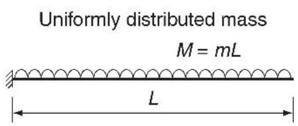
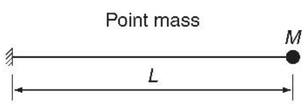
M_{pm}: Ultimate sagging moment capacity within span

v_1 and v_2: Ultimate shear stress at distance d_e from face of left- and right-hand supports respectively

V_{s1} and V_{s2}: Dynamic reaction at left- and right-hand ends respectively

[c] Valid only if $M_{ps} = M_{pm}$

Equivalent SDOF Factors for Cantilevers

Loading diagram	Mass diagram	Strain range	Load factor, K_L	Mass factor, K_M	Load–mass factor, K_{LM}	Maximum resistance, R_m	Stiffness, k	Support shear, V_s	Ultimate shear stress, v	Dynamic reaction, V
		Elastic	0.40	0.26	0.65	$\frac{2M_p}{L}$	$\frac{8EI}{L^3}$	R_m	$R_m\left(\frac{1}{d_e}-\frac{1}{L}\right)$	$0.69R + 0.31F$
		Plastic	0.50	0.33	0.66	$\frac{2M_p}{L}$	0			$0.75R_m + 0.25F$
		Elastic	0.40	1.0	2.5	$\frac{2M_p}{L}$	$\frac{8EI}{L^3}$	R_m	$R_m\left(\frac{1}{d_e}-\frac{1}{L}\right)$	$0.50R + 0.50F$
		Plastic	0.50	1.0	2.0	$\frac{2M_p}{L}$	0			$0.50R_m + 0.50F$
		Elastic	1.0	0.24	0.24	$\frac{M_p}{L}$	$\frac{3EI}{L^3}$	R_m	$\frac{R_m}{d_e}$	$1.74R - 0.74F$
		Plastic	1.0	0.33	0.33	$\frac{M_p}{L}$	0			$1.50R_m - 0.50F$
		Elastic	1.0	1.0	1.0	$\frac{M_p}{L}$	$\frac{3EI}{L^3}$	R_m	$\frac{R_m}{d_e}$	$1.0R$
		Plastic	1.0	1.0	1.0	$\frac{M_p}{L}$	0			$1.0R_m$

Notation

M_p: Ultimate hogging moment capacity at support

Equivalent SDOF for Two-Way Slabs: Simple Supports

Strain range	a/b	Load factor, K_L	Mass factor, K_M	Load–mass factor, K_{LM}	Maximum resistance, R_m	Stiffness, k	Dynamic reaction, V: V_A	Dynamic reaction, V: V_B
Elastic	1.0	0.46	0.31	0.67	$\frac{12}{a}(M_{pa}+M_{pb})$	$\frac{252EI}{a^2}$	$0.07F+0.18R$	$0.07F+0.18R$
	0.9	0.47	0.33	0.70	$\frac{1}{a}(12M_{pa}+11M_{pb})$	$\frac{230EI}{a^2}$	$0.06F+0.16R$	$0.08F+0.20R$
	0.8	0.49	0.35	0.71	$\frac{1}{a}(12M_{pa}+10.3M_{pb})$	$\frac{212EI}{a^2}$	$0.06F+0.14R$	$0.08F+0.22R$
	0.7	0.51	0.37	0.73	$\frac{1}{a}(12M_{pa}+9.8M_{pb})$	$\frac{201EI}{a^2}$	$0.05F+0.13R$	$0.08F+0.24R$
	0.6	0.53	0.39	0.74	$\frac{1}{a}(12M_{pa}+9.3M_{pb})$	$\frac{197EI}{a^2}$	$0.04F+0.11R$	$0.09F+0.26R$
	0.5	0.55	0.41	0.75	$\frac{1}{a}(12M_{pa}+9.0M_{pb})$	$\frac{201EI}{a^2}$	$0.04F+0.09R$	$0.09F+0.28R$
Plastic	1.0	0.33	0.17	0.51	$\frac{12}{a}(M_{pa}+M_{pb})$	0	$0.09F+0.16R_m$	$0.09F+0.18R_m$
	0.9	0.35	0.18	0.51	$\frac{1}{a}(12M_{pa}+11M_{pb})$	0	$0.08F+0.15R_m$	$0.09F+0.18R_m$
	0.8	0.37	0.20	0.54	$\frac{1}{a}(12M_{pa}+10.3M_{pb})$	0	$0.07F+0.13R_m$	$0.10F+0.20R_m$
	0.7	0.38	0.22	0.58	$\frac{1}{a}(12M_{pa}+9.8M_{pb})$	0	$0.06F+0.12R_m$	$0.10F+0.22R_m$
	0.6	0.40	0.23	0.58	$\frac{1}{a}(12M_{pa}+9.3M_{pb})$	0	$0.05F+0.10R_m$	$0.10F+0.25R_m$
	0.5	0.42	0.25	0.59	$\frac{1}{a}(12M_{pa}+9.0M_{pb})$	0	$0.04F+0.08R_m$	$0.11F+0.27R_m$

Notation

M_{pa}: Total positive ultimate moment capacity along midspan section parallel to short edge
M_{pb}: Total positive ultimate moment capacity along midspan section parallel to long edge
F: Force
k: Stiffness
R: Resistance
R_m: Maximum resistance
V_A: Total dynamic reaction along short edge
V_B: Total dynamic reaction along long edge
Note: F, k, R and R_m refer to the *total* load on the slab